Reasoning about Space and Change with Answer Set Programming Modulo Theories

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ASPMT (QS)

Motivations

My work on the PhD thesis concerns nonmonotonic reasoning about relations between spatial objects and the way they change in time.

Reasoning about space and the way objects and spatial relations can change is a key element in systems that aim at modelling a wide range of dynamic application domains, e.g., in robotics or spatial planning, where tasks like causal explanation and default reasoning often need to be considered mutually with spatial consistency. Therefore my aim is to introduce a **computational framework** that enables to perform nonmonotonic spatial reasoning (dealing with default rules, frame problem, indirect effect, etc.) that **may be used in practical applications**.

Accomplished Work

The work I have accomplished so far consists in a collaborated research with Mehul Bhatt and Carl Schultz which resulted in establishing the **ASPMT(QS) system [1] which is a novel approach for reasoning about spatial change within a KR paradigm**. ASPMT(QS) is based on a paradigm of Answer Set Programming Modulo Theories (ASPMT) [2] and polynomial encodings of spatial relations. The system is capable of sound and complete spatial reasoning, and combining qualitative and quantitative spatial information when reasoning nonmonotonically. Its first version is already implemented.

We have demonstrated (see [1]) that no other existing spatial reasoning system is capable of supporting the key nonmonotonic spatial reasoning features (e.g., spatial inertia, ramification) provided by ASPMT(QS) in the context of a mainstream knowledge representation and reasoning method, namely, answer set programming.

The system builds on ASPMT2SMT [2] – a compiler translating a tight fragment of ASPMT into SMT instances. Our system consists of an additional module for spatial reasoning and Z3 [4] as the SMT solver. A minimal prototypical implementation of ASPMT(QS) is available online from Docker Hub: https://hub.docker.com/r/spatialreasoning/aspmtqs/.

Qualitative Space

Basic **domain entities** in qualitative space with polynomial encodings include *circles, triangles, points* and *segments*:

- a *point* is a pair of reals x, y
- a *line segment* is a pair of end points p_1, p_2 $(p_1 \neq p_2)$
- a *circle* is a centre point p and a real radius r $(0 < r)$
- a *triangle* is a triple of vertices (points) p_1, p_2, p_3 such that p_3 is *left of* segment p_1, p_2.

We define a range of **spatial relations** with the corresponding polynomial encodings, e.g.,

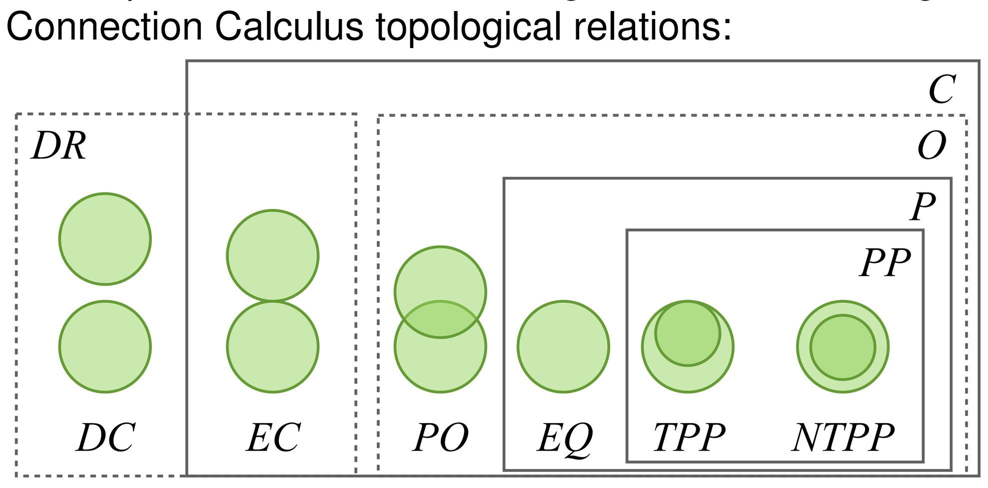
- Relative orientation relations, e.g., *left, right, collinear*, orientation relations between *points* and *segments*, and *parallel, perpendicular* relations between *segments*,
- Mereotopology relations, e.g., *Part-whole* and *contact* relations between regions.

The representation is expressive enough to cover a number of other relations known from the literature:

Theorem. *ASPMT(QS) is capable to express relations of:*

- *Interval Algebra [5],*
- *Rectangle Algebra [6],*
- *Region Connection Calculus [7],*
- *Cardinal Direction Calculus [8].*

Our representation enables, e.g., to define all Region Connection Calculus topological relations:



where the RCC–8 base relations are:

- DC – disconnected,
- EC – externally connected,
- EQ – equal,
- PO – partially overlapping,
- TPP – tangential proper part,
- $TPPi$ – tangential proper part inverse,
- $NTPP$ – non-tangential proper part,
- $NTPPi$ – non-tangential proper part inverse.

ASPMT(QS) Program

The **input** program is divided into:

- `sorts` (data types),
- `objects` (particular elements of given types),
- `constants` (functions),
- `variables` (variables associated with declared types).

The second part of the program consists of clauses.

ASPMT(QS) supports:

- connectives: `&`, `|`, `not`, `->`, `<-`,
- arithmetic operators: `<`, `<=`, `>=`, `>`, `=`, `!=`, `+`, `=`, `*`,
- `sorts` for geometric objects types, e.g., `point`, `segment`, `circle`, `triangle`,
- functions describing objects parameters, e.g., $x(\mathtt{point})$, $r(\mathtt{circle})$,
- qualitative spatial relations, e.g., `rccEC(circle, circle)`, `coincident(point, circle)`.

The **output**:

a stable model (see [3]) of the input program, or a statement that no such model exists.

Example

Topological information about circles a, b, c:

- a is a proper part of b,
- b is discrete from c,
- a is in contact with c.

Input program:

```
:- constants
a  :: circle;
b  :: circle;
c  :: circle.

<- not rccPP(a,b).
<- not rccDR(b,c).
<- not rccC(a,c).
```

Output:

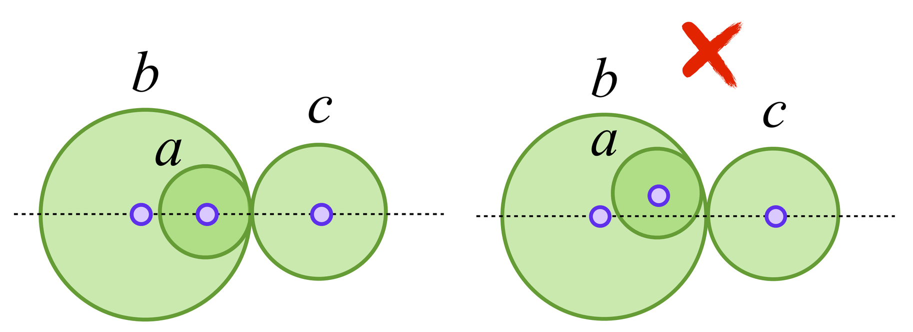
```
r(a) = 1.0              r(b) = 2.0             r(c) = 1.0
x(a) = 1.0              x(b) = 0.0             x(c) = 3.0
y(a) = 0.0              y(b) = 0.0             y(c) = 0.0
rccTPP(a,b) = true      rccEC(a,c) = true      rccEC(b,c) = true
```

Addition to the **input** program:

```
<- not left_of(ax,ay,bx,by,cx,cy)=true.
```

Output of the extended program:

```
UNSATISFIABLE;
```



ASPMT(QS) refines the topological relations to infer that:

- a must be a *tangential proper part* of b,
- both a and b must be *externally connected* to c.

Example

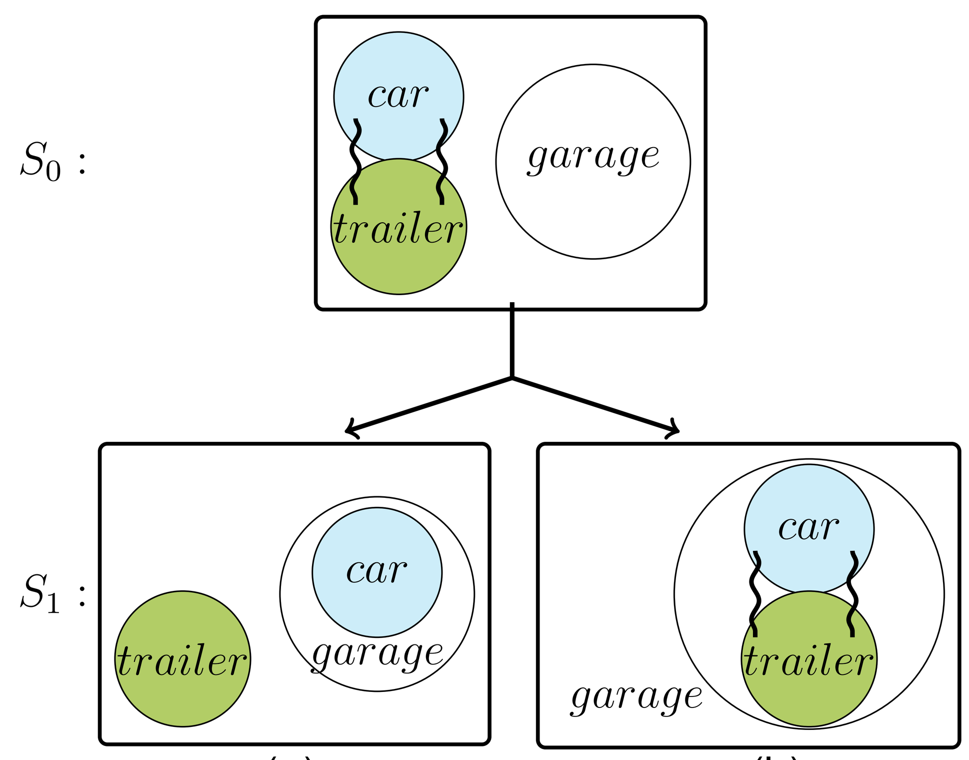
In S_0 the car is attached to the $trailer$ and they are outside the $garage$. In S_1, the car is inside the $garage$. What actions have been performed if by default the $trailer$ is attached to the car?

Allowed domain-specific actions:

- the car can move,
- the $trailer$ can be detached.

Attachment I. Given the topological information in S_0, ASPMT(QS) infers that (b) the car, together with the $trailer$ move into the $garage$.

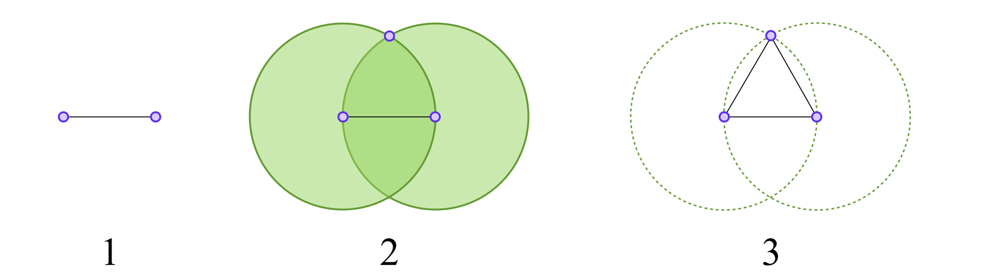
Attachment II. Given additional geometric information: $r(car) = 2$, $r(trailer) = 2$ and $r(garage) = 3$, ASPMT(QS) infers that (b) is now inconsistent, and the only possible solution is (a).



(a)   (b)

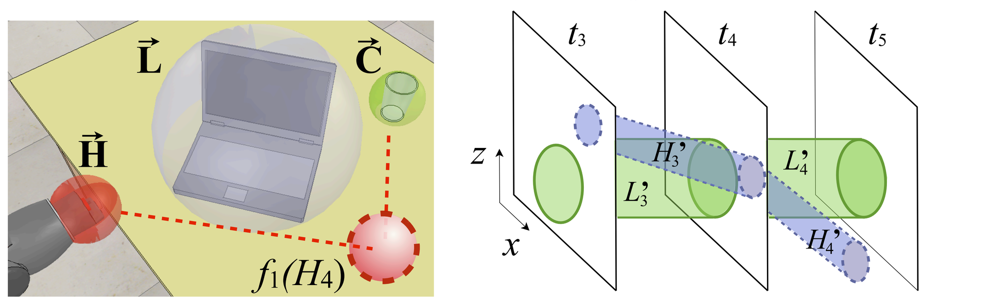
Applications:

Validity of Euclid Constructions:



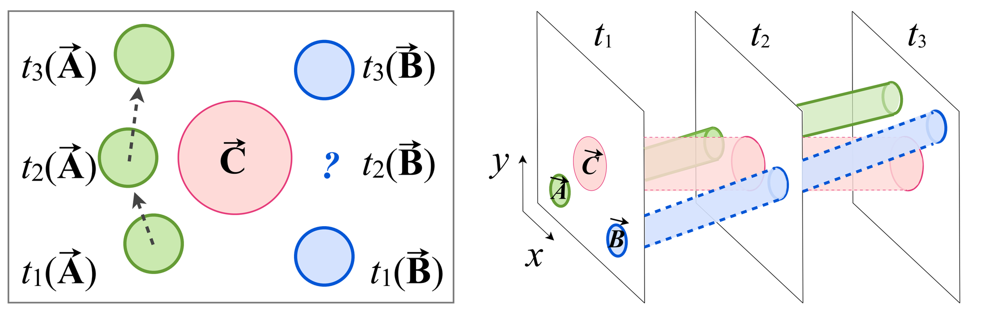
Does the Euclid construction (1–3) enable to construct an equilateral triangle? Is the constructed triangle always equilateral?

Motion Planning:



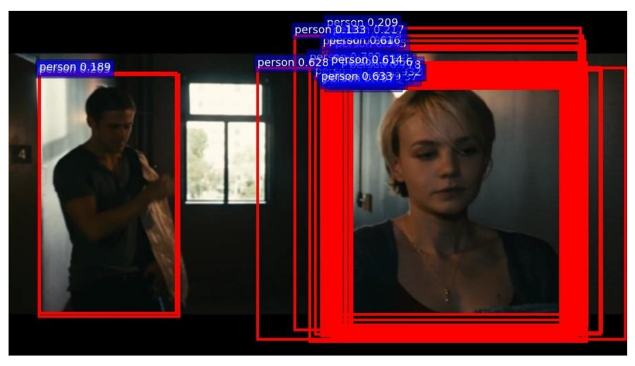
Plan motions of a robotic arm in order to get the cup of coffee without the risk of spilling the coffee in presence of limited range of motions.

Abduction of Robots Position:



How a robot A should infer position of a robot B at a time-point t_2 based on partial observations and minimization of spatial change?

People Tracking:



(1)

(2)

(3)

What has happened to the object on the left between frames 1 and 2?

Future Work

We plan to:

- extend the ASPMT(QS) system to enable performing more complex spatio-temporal reasoning,
- apply the system to further practical problems such as computer-aided architecture design, mobile robots control, etc.

References

[1] P. A. Wałęga, M. Bhatt, and C. Schultz, "Aspmt (qs): Non-monotonic spatial reasoning with answer set programming modulo theories," in *Logic Programming and Nonmonotonic Reasoning*. Springer, 2015, pp. 488–501.

[2] M. Bartholomew and J. Lee, "System aspmt2smt: Computing ASPMT Theories by SMT Solvers," in *Logics in Artificial Intelligence*. Springer, 2014, pp. 529–542.

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[5] J. F. Allen, "Maintaining knowledge about temporal intervals," *Communications of the ACM*, vol. 26, no. 11, pp. 832–843, 1983.

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