# Nonmonotonic Qualitative Spatial Reasoning

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## Motivations

My work on the PhD thesis concerns nonmonotonic reasoning about relations between spatial objects and the way they change in time.

My **motivations** are two-folded:

- Try to **model human reasoning** about changing spatial configurations. Humans methods are surprisingly accurate while reasoning without complete or precise information. Modelling such methods may help to understand human spatial reasoning and introduce better AI approaches for spatial reasoning.
- Reasoning about space and the way objects and spatial relations can change is a key element in systems that aim at modelling a wide range of dynamic application domains, e.g., in robotics or spatial planning, where tasks like causal explanation and default reasoning often need to be considered mutually with spatial consistency. Therefore my aim is to introduce a **computational framework** that enables to perform nonmonotonic spatial reasoning (dealing with default rules, frame problem, indirect effect, etc.) that **may be used in practical applications**.

## Accomplished Work

The **work accomplished** so far amounts to constructing:

- **HLQL** – Hybrid Logic for Qualitative Reasoning about Location – the only modal logic that enables to reason about subject-oriented directional relations with respect to other objects;
- **ASPMT(QS)** – a general framework for spatial reasoning within the paradigm of Answer Set Programming Modulo Theories – the only computational framework that enables to perform nonmonotonic reasoning about spatial relations by means of stable model semantics [1].

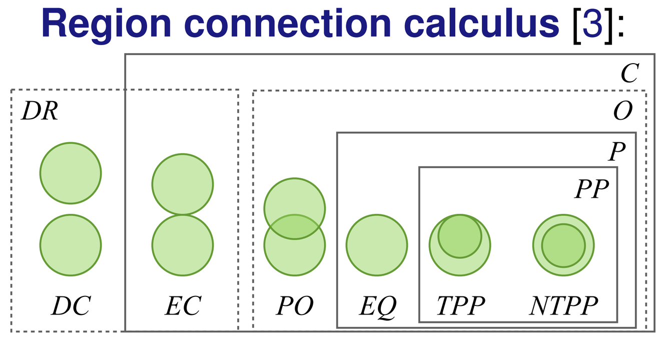
## Qualitative Space

Qualitative spatial representation and reasoning [2] involve (mainly) qualitative calculi (relational algebras) and modal logics, e.g. :

**Qualitative calculi:**

- interval algebra,
- rectangle algebra,
- region connection calculus,
- cardinal direction calculus.

**Region connection calculus** [3]:




RCC–8 base relations:

- $DC$ – disconnected,
- $EC$ – externally connected,
- $EQ$ – equal,
- $PO$ – partially overlapping,
- $TPP$ – tangential proper part,
- $TPPi$ – tangential proper part inverse,
- $NTPP$ – non-tangential proper part,
- $NTPPi$ – non-tangential proper part inverse.

**Modal logics:**

- compass logic,
- spatial propositional neighbourhood logic,
- cone logic.

**Compass logic** [4]:

- considers points in 2D space,
- uses 2 irreflexive linear orders for 2 Cartesian coordinates:

  $\langle T_1, <_1 \rangle$ for "lying horizontally",

  $\langle T_2, <_2 \rangle$ for "lying vertically",
- structures are of a form $\mathcal{T} = \langle T_1, <_1, T_2, <_2, V \rangle$, where $V : \langle T_1 \times T_2 \rangle \to \mathcal{P}(Var)$,
- 4 modalities:

  $\Diamond_{\uparrow}$ for "increases $y$",

  $\Diamond_{\downarrow}$ for "decreases $y$",

  $\Diamond_{\rightarrow}$ for "increases $x$",

  $\Diamond_{\leftarrow}$ for "decreases $x$".

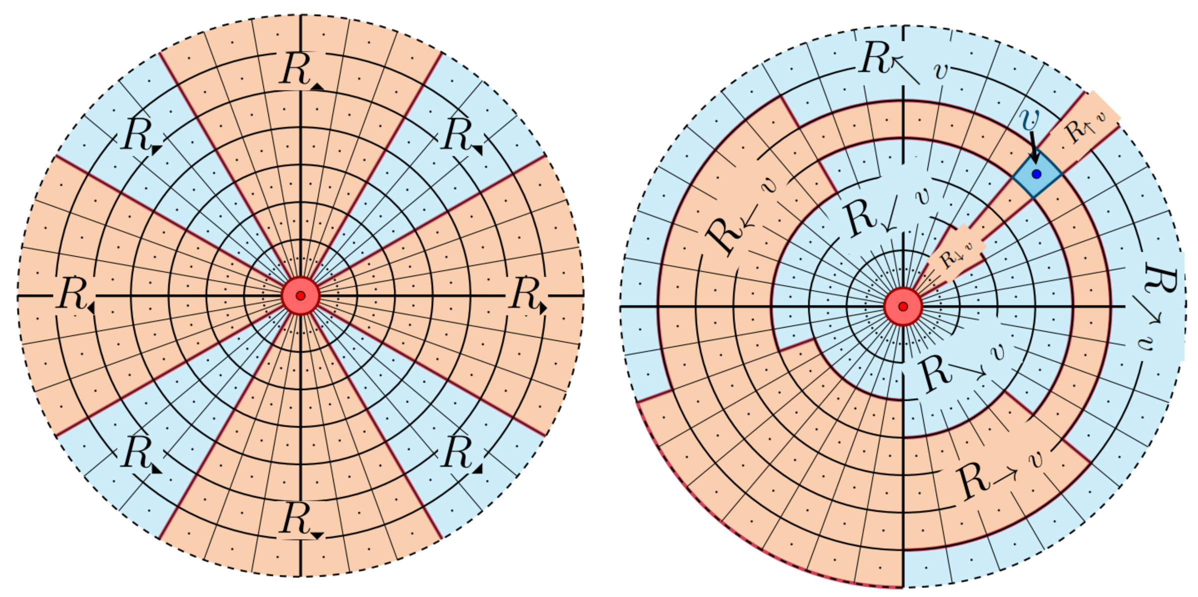
## HLQL – joint work with M. Zawidzki and T. Lechowski

Hybrid Logic for Qualitative Reasoning about Location (HLQL) [5] is a multimodal logic for qualitative reasoning about location of objects in a flat **2-dimensional space**. The considered spatial relations are **subject-oriented**, i.e., they capture space from the subject's point and **not from the aerial point** of view as most of spatial logics do (e.g., compass logic, spatial propositional neighbourhood logic and cone logic).

The logic enables to represent relations asserting **mutual objects' locations with respect to each other, seen from subject's perspective**, which to the best of our knowledge have not been previously discussed in the literature.

## Space Representation in HLQL

- The central locus of the space is occupied by a **subject**, faced upwards;



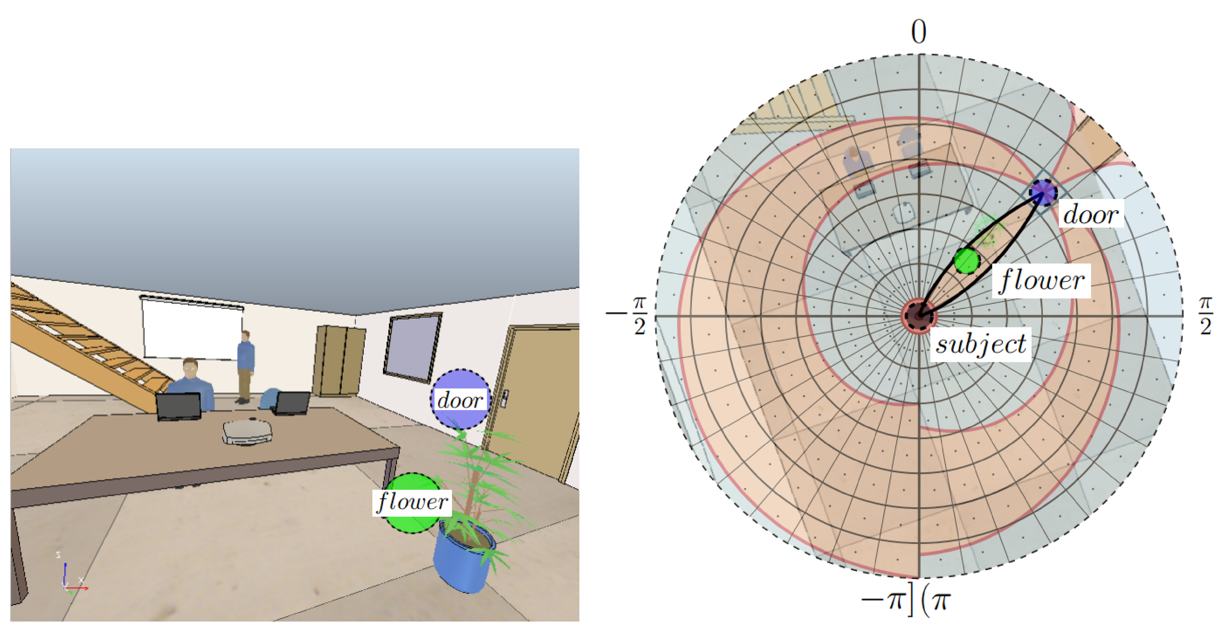
(a) (b)

(a) 8 directional relations indicating the **location of an object with respect to the subject**: $R_{\blacktriangleleft}$ – *to the right*, $R_{\blacktriangledown}$ – *behind*, $R_{\blacktriangle}$ – *in front of*, etc.;

(b) 8 relations of **other objects' locations with respect to another object** $v$: $R_{\leftarrow v}$ – *to the left of* $v$, $R_{\rightarrow v}$ – *to the right of* $v$, $R_{\downarrow v}$ – *in front of* $v$, etc.

## Modeling Humans Cognition in HLQL

HLQL allows to model human-like **subject-oriented** representation of space:




(a) (b)

(a) From subject's point of view "the flower is in front of the door".

(b) In HLQL we have $R_{\downarrow}(flower, door)$ which has a desired intuitive meaning that "the flower is in front of the door from subject's point of view".

## ASPMT(QS) – joint work with M. Bhatt and C. Schultz

ASPMT(QS) [6] is a computational framework for spatial reasoning within the paradigm of Answer Set Programming Modulo Theories. The system builds on ASPMT2SMT [7] – a compiler translating a tight fragment of ASPMT into SMT instances. Our system consists of an additional module for spatial reasoning and Z3 [8] as the SMT solver. A minimal prototypical implementation of ASPMT(QS) is available online from Docker Hub: `https://hub.docker.com/r/spatialreasoning/aspmtqs/`.

The **input** program is divided into:

- `sorts` (data types),
- `objects` (particular elements of given types),
- `constants` (functions),
- `variables` (variables associated with declared types).

The **output** is a **stable model** of the input program.

## Topology and Orientation in ASPMT(QS)

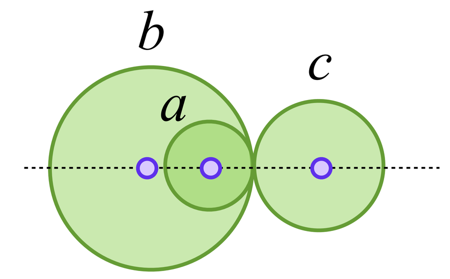
**Input** program:

```
:- constants
a  :: circle;
b  :: circle;
c  :: circle.

<- not rccPP(a,b).
<- not rccDR(b,c).
<- not rccC(a,c).
```

**Output**:

```
r(a) = 1.0              r(b) = 2.0            r(c) = 1.0
x(a) = 1.0              x(b) = 0.0            x(c) = 3.0
y(a) = 0.0              y(b) = 0.0            y(c) = 0.0
rccTPP(a,b) = true      rccEC(a,c) = true     rccEC(b,c) = true
```

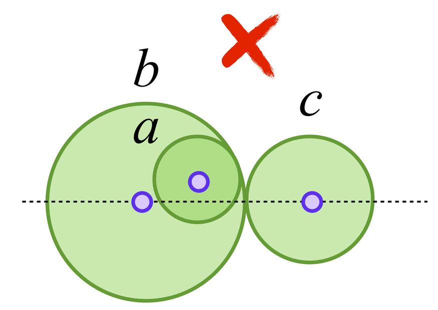



Addition to the **input** program:

```
<- not left_of(ax,ay,bx,by,cx,cy)=true.
```

**Output** of the extended program:

```
UNSATISFIABLE;
```




## Spatial Frame Problem in ASPMT(QS)

In $S_0$ the $car$ is attached to the $trailer$ and they are outside the $garage$. In $S_1$, the $car$ is inside the $garage$. What actions have been performed?

Allowed domain-specific actions:

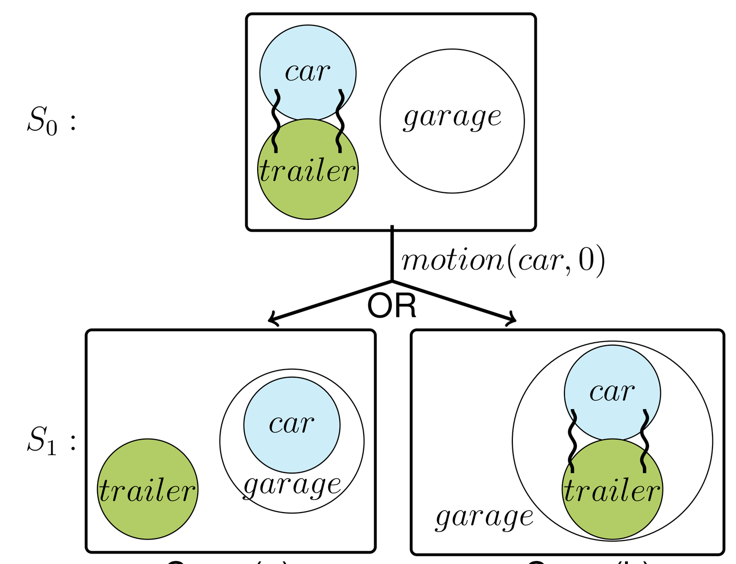
- the $car$ can move: $\mathtt{move}(car, X)$,
- the $trailer$ can be detached: $\mathtt{detach}(car, trailer, X)$.

**Attachment I.** Given the topological information in $S_0$, ASPMT(QS) infers that there are two possible solutions:

(a) the $car$ is detached from the $trailer$,

(b) the $car$, together with the $trailer$ move into the $garage$.

**Attachment II.** Given additional geometric information: $r(car) = 2$, $r(trailer) = 2$ and $r(garage) = 3$, ASPMT(QS) infers that (b) is now inconsistent, and the only possible solution is (a).




Case (a) Case (b)

## Future Work

- **HLQL**: conduct deeper research on similarities and differences between the introduced representation methods and those *actually* used by humans. In other words, check cognitive adequacy of the approach.
- **ASPMT(QS)**: encode more complex spatial and spatio-temporal relations to reason, e.g., about the so-called spatio-temporal-histories, i.e., objects that continuously change over time. Moreover, apply ASPMT(QS) for such practical problems as computer-aided architecture design and mobile robots control.
- **Preferred mental models**: introduce logical methods for computing the so called preferred mental models that are constructed by humans while reasoning about spatial and temporal information when a number of various models are consistent with the description.

## References

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